Quantum stabilization of dilatonic Anti-de Sitter Universe

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The possibility of quantum creation of a dilatonic AdS Universe is discussed.Without dilaton it is known that quantum effects lead to the annihilation of AdS Universe. We consider the role of the form for the dilatonic potential in the quantum creation of a dilatonic AdS Universe. Using the conformal anomaly for dilaton coupled scalar, the anomaly induced action and the equations of motion are obtained. The anomaly induced action is added to classical dilaton gravity action. The solutions of the full theory which correspond to quantum-corrected AdS Universe are given for number of dilatonic potentials.

1. Introduction

There is much interest now to various studies of Anti-de Sitter (AdS) space. This is motivated by AdS/CFT correspondence (for a review, see [1]) according to which (in its simplest form), the properties of classical AdS space may correspond to the ones of some dual conformal field theory in less dimensions.

From another side, quantum field theory in curved spacetime (see [2], for a review) may help in our understanding of the origin of the early Universe. In particular, one of most successful inflationary models (so called trace anomaly induced inflation [6]) is based on quantum creation of de Sitter Universe by matter quantum effects. It is very interesting to understand if this is specific property of de Sitter space or it is quite general phenomenon?

In this work, we consider the role of dilaton coupled quantum matter to stabilization of AdS Universe. It is known that without dilaton, quantum effects destabilize and annihilate AdS Universe [8]. These quantum effects in our work are accounted for by four-dimensional dilaton conformal anomaly (for a review, see [7]). It is very interesting to note that such dilaton conformal anomaly in $N = 4$ super Yang-Mills theory with $N = 4$ conformal supergravity may have the holographic origin (via AdS/CFT) as it was shown in [11].

Using anomaly induced effective action for dilaton coupled scalar and adding it to the classical gravity action, we analyze the equations of motion, for a number of dilatonic potentials. It is shown that the possibility of quantum creation of dilatonic AdS space occurs.

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2. Description of the Model

First of all we will consider quantum fields in AdS space. At the end of this section we will also review quantum annihilation of Anti-de Sitter Universe [8].

Our model is given as a 4-dimensional Anti-de Sitter spacetime (AdS_4) and the metric is chosen to be [8]

$$ds^2 = e^{-2\lambda\tilde{x_3}}(dt^2 - (dx^1)^2 - (dx^2)^2) - (d\tilde{x}^3)^2 , \quad (2.1)$$

with a negative effective cosmological constant $\Lambda = -\lambda^2$. We may rewrite this metric in conformally flat form by using the following transformation

$$y = x^3 = \frac{e^{-\lambda\tilde{x_3}}}{\lambda}. \quad (2.2)$$

taking $a = e^{-\lambda\tilde{x_3}} = 1/(\lambda x^3) = 1/(\lambda y)$, we finally obtain for the metric the following expression

$$ds^2 = a^2(dt^2 - dx^2) = a^2\eta_{\mu\nu}dx^\mu dx^\nu. \quad (2.3)$$

It is important to remark that the obtained form for the metric is the most useful in the study of quantum gauge theory via the SG dual.

Now, let us imagine that the early Universe is filled by some grand unified theory (GUT). In that case, it is known [2] that it is enough to consider only free fields, as radiative corrections are not essential. It is also important to remark that unlike to the de Sitter space the AdS space is supersymmetric background for GUT in the case if it is SUSY. For a review of the quantum fields on negative curvature space see ref. [3].

In order to construct the effective action in terms of the quantum scalar field theory, (for simplicity) we will consider the anomaly, expressed as (for a review see [4, 7])

$$T = b\left(F+\frac{2}{3}\Box R\right)+b'G+b''\Box R + +a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}+a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right), \quad (2.4)$$

where for single scalar,

$$b = \frac{1}{120(4\pi)^2}, \quad b' = -\frac{1}{120(4\pi)^2}, \quad a_1 = \frac{1}{32(4\pi)^2}, \quad a_2 = \frac{1}{24(4\pi)^2}, \quad (2.5)$$

while F is the square of the Weyl tensor in four dimensions , and G is the Gauss-Bonnet invariant

$$F = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-2R_{\mu\nu}R^{\mu\nu}+\frac{1}{3}R^2, \quad (2.6)$$

Note that the constant b'' is an arbitrary parameter (it can be changed by a finite renormalization of the gravitational action). As the physical meaning does not change we may put $b''=0$. The terms with coefficients a_1 and a_2 correspond to the contribution of the dilaton, where f is an arbitrary dilatonic coupling $f=f(\varphi)$. Their contribution to four dimensional conformal anomaly is found in ref.[10]. The initial action for dilaton coupled conformal quantum scalar is given by:

$$S=\int d^4x\sqrt{-g}\phi f(\Box-\frac{1}{6}R)\phi, \quad (2.7)$$

where ϕ is quantum scalar filed.

Note also that if in (2.4) the terms, which contain dilaton, are omitted we obtain the well known conformal anomaly, for scalar field.

3. Effective action and equations of motion

By using this conformal anomaly it is not difficult to construct the anomaly-induced effective action [5]. Taking into account that it is useful to consider a conformal metric for a AdS space, one may write the metric for our case as $g_{\mu\nu}=e^{2\sigma(y)}\eta_{\mu\nu}$, which is similar to (2.3), where $\eta_{\mu\nu}$ is the Minkowski metric. After this, using the techniques of ref. [5], the following anomaly-induced effective action on AdS space is obtained,

$$W=\int d^4x\Big\{2b_1\sigma\Box^2\sigma-\frac{1}{12}\left(b''+\frac{2}{3}(b+b')\right)\Big(6\Box\sigma + +6\eta^{\mu\nu}(\partial_\mu\sigma)(\partial_\nu\sigma)\Big)^2+a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}\sigma + +a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)\sigma+a_2\frac{(\nabla f)(\nabla f)}{f^2}[(\nabla\sigma)(\nabla\sigma)]\Big\}.$$

Since σ depends only on y and supposing that $f=\varphi$, this expression may be simplified, and finally the following expression is obtained for the effective action:

$$W= V_3\int dy\Big[2b_1\sigma\sigma''''-3\big(b''+\frac{2}{3}(b+b')\big)(\sigma''+(\sigma')^2)^2+ + a_1\frac{(\varphi')^4}{\varphi^4}\sigma+a_2\left[\frac{(\varphi')^2}{\varphi^2}\right]''\sigma+a_2\frac{(\varphi')^2}{\varphi^2}(\sigma')^2\Big]. \quad (3.8)$$

In the last equation we have $\sigma'=d\sigma/dy$. On the other hand it is well known that the total effective action consists of W plus some conformally invariant functional. But, since we are considering a conformally flat background, this conformally invariant is a non-essential constant. It could become more important, similar to a kind of Casimir energy, if we considered periodicity on some of the coordinates (say, AdS BH) because in such a situation it would depend on the radius of the compact dimension.

The quantum matter effects in the AdS Universe may be accounted by adding the anomaly-induced action to the classical gravitational action, which in this case includes a dilatonic potential $V(\varphi)$ and a kinetic term for the dilaton, and has the following form,

$$\begin{aligned} S_{\rm cl} &= -\frac{1}{\kappa}\int d^4x\sqrt{-g}\,(R+\frac{\beta}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+V(\varphi)+ \\ &\quad +6\Lambda)= \\ &= -\frac{1}{\kappa}\int d^4x\, e^{4\sigma}(6e^{-2\sigma}((\sigma')^2+(\sigma''))+ \\ &\quad +\frac{\beta}{2}e^{2\sigma}(\varphi')^2+V(\varphi)+6\Lambda). \end{aligned} \quad (3.9)$$

Now, in order to describe the dynamics of the whole quantum system, we must take the sum of the classical action and the quantum effective action.

Varying the action $S_{cl}+W$ with respect to σ and φ, one may obtain the equations of motion, assuming in this case that σ and φ depends only on conformal coordinate y, we finally arrive at the following equations of motion

$$\begin{aligned} &a_1\frac{(\varphi')^4}{\varphi^4}+2a_2[\frac{(\varphi'')^2}{\varphi^2}+\frac{\varphi'\varphi'''}{\varphi^2}-5\frac{(\varphi')^2\varphi''}{\varphi^3}+ \\ &+\ 3\frac{(\varphi')^4}{\varphi^4}-\frac{a''}{a}\frac{(\varphi')^2}{\varphi^2}+\frac{(a')^2}{a^2}\frac{(\varphi')^2}{\varphi^2}-2\frac{a'}{a}\frac{\varphi'\varphi''}{\varphi^2}+ \\ &+\ 2\frac{a'}{a}\frac{(\varphi')^3}{\varphi^3}]-4b[\frac{a''''}{a}-4\frac{a'a'''}{a^2}+6\frac{(a')^2a''}{a^3}+ \\ &-\ 3\frac{(a'')^2}{a^2}]+24b'\left[\frac{(a')^2a''}{a^3}-\frac{(a')^4}{a^4}\right]-\frac{24}{\kappa}a^4\Lambda- \\ &-\ \frac{12}{\kappa}aa''+3\beta a^6(\varphi')^2+4a^4V(\varphi)=0 \end{aligned}$$

$$\begin{aligned} 6\quad &a_1\ln a\left[\frac{(\varphi')^4}{\varphi^5}-\frac{(\varphi')^2\varphi''}{\varphi^4}\right]-2a_1\frac{a'}{a}\frac{(\varphi')^3}{\varphi^4}+ \\ &+\ a^4\frac{\partial V(\varphi)}{\partial\varphi}-\frac{\beta}{2}a^6\varphi''-3\beta a^6\frac{a'}{a}\varphi'+a_2[\frac{a'a''}{a^2}\frac{\varphi'}{\varphi^2}+ \\ &+\ \frac{a''}{a}\left(\frac{(\varphi')^2}{\varphi^3}-\frac{\varphi''}{\varphi^2}\right)-\frac{\varphi'}{\varphi^2}\frac{a'''}{a}]=0 \end{aligned} \quad (3.10)$$

Here prime means derivative with respect to y. In order to analyze these equations it is interesting to consider the more simple case, when there is no dilaton. In such a situation from equations (3.10) it is very easy to obtain the following equation of motion,

$$\frac{a''''}{a} - 4\frac{a'a'''}{a^2} - 3\frac{(a'')^2}{a^2} + \left(6 - 6\frac{b'}{b}\right)\frac{a''(a')^2}{a^3} + \\ + \frac{6b'(a')^4}{ba^4} - \frac{a}{4bk}\Big(- 12a'' - 24\Lambda a^3\Big) = 0. \qquad (3.11)$$

So as a result we have obtained the same equation, which is discussed in [8] in the similar situation when the terms depending on the dilaton are not present. Doing the same analysis, as in that reference, we see that looking for the special AdS-like solutions of (3.11) in the form: $a = c/y$, when there are no quantum corrections, there is a solution with $c = 1/\sqrt{-\Lambda}$, in accordance with the AdS metric. On the other hand if the effective cosmological constant $\Lambda = 0$, Eq. (3.11) reduces to $c^2 = b'\kappa$ (at $a(y) = c/y$). However, $c^2 = b'\kappa$ leads to an imaginary scale factor a, because $b' < 0$. We must understand by this that an Anti-de Sitter Universe can not be created just as a result of matter quantum corrections. But we know that there exists the possibility of creation of a de Sitter Universe by solely matter quantum effects [6]. Truly, for a scale factor, which depends only on time, the sign of curvature (which is positive) is changing. As a result, $a = c/\eta$ with $c^2 = -b'\kappa$. So we conclude that, there is always a solution which leads to a quantum created de Sitter Universe. On the contrary, we see that a necessary condition for the existence of an Anti-de Sitter Universe is the presence, in classical theory, of a negative effective cosmological constant.

In the general case for c^2 (assuming $\Lambda < 0$) it is easily obtained the following algebraic equation :

$$\kappa b' - c^2 - \Lambda c^4 = 0 \;, \qquad (3.12)$$

and its solutions have the form:

$${c_1}^2 = -\frac{1}{2\Lambda}\left(1 + \sqrt{1 + 4\kappa b'\Lambda}\right) \qquad (3.13)$$

and

$${c_2}^2 = -\frac{1}{2\Lambda}\left(1 - \sqrt{1 + 4\kappa b'\Lambda}\right). \qquad (3.14)$$

From these solutions we see that in the first of them, if we start from some bare (very small) negative cosmological constant, we get an Anti-de Sitter Universe with a smaller cosmological constant due to quantum corrections. So we find that the quantum corrections act against the existing Anti-de Sitter Universe and make it less stable. This is the mechanism of annihilation of Anti-de Sitter Universe [8].

The second solution from the physical point of view it is not interesting because it corresponds to the imaginary scale factor since in it $c^2 < 0$.

With the solution like $a(y) = c/y$ it is possible to obtain the the dependence of the complete effective action on c^2, and we get for it,

$$S_{\rm cl} + W = \\ = V_3 \int \frac{{\rm d}y}{y^4}\left[6b'\ln\left(\frac{c^2}{y^2}\right) - 8(b + b') - \frac{6(\Lambda c^4 - c^2)}{\kappa}\right] \\ = V_3 \int \frac{{\rm d}y}{y^4}\left[6b'\ln\left(\frac{c^2}{y^2}\right) - 8b - 14b' + \frac{12c^2}{\kappa}\right]. \qquad (3.15)$$

Summing, because of quantum corrections an already existing Anti-de Sitter Universe becomes less stable, whereas creation of an Anti-de Sitter Universe by solely quantum corrections as we have seen is impossible. Now, one may wonder which effects can be obtained if we include in our scenario the terms with the dilaton.

4. Dilaton coupled theories and stabilization of Anti-de Sitter Universe

As examples of theories where dilaton appears we may consider the equations (3.10) and look for their solutions for different dilatonic potentials. It is very interesting to think about the most common functions for the dilatonic potential (like in the superstring theories) and look for the possibility of the creation of an Anti-de Sitter Universe in such a gravitational background. In the following, for simplicity, we will consider the case when the kinetic term for the dilaton is absent.

1. Firstly let us see the case when in (3.9) the dilatonic potential is not present. In this situation the equations of motion (3.10) take the form,

$$a_1\frac{(\varphi')^4}{\varphi^4} + 2a_2[\frac{(\varphi'')^2}{\varphi^2} + \frac{\varphi'\varphi'''}{\varphi^2} - 5\frac{(\varphi')^2\varphi''}{\varphi^3} + \\ + 3\frac{(\varphi')^4}{\varphi^4} - \frac{a''}{a}\frac{(\varphi')^2}{\varphi^2} + \frac{(a')^2}{a^2}\frac{(\varphi')^2}{\varphi^2} - 2\frac{a'}{a}\frac{\varphi'\varphi''}{\varphi^2} + \\ + 2\frac{a'}{a}\frac{(\varphi')^3}{\varphi^3}] - 4b[\frac{a''''}{a} - 4\frac{a'a'''}{a^2} + 6\frac{(a')^2a''}{a^3} + \\ - 3\frac{(a'')^2}{a^2}] + 24b'\left[\frac{(a')^2a''}{a^3} - \frac{(a')^4}{a^4}\right] - \frac{24}{\kappa}a^4\Lambda - \\ - \frac{12}{\kappa}aa'' = 0$$

$$6a_1\ln a\left[\frac{(\varphi')^4}{\varphi^5} - \frac{(\varphi')^2\varphi''}{\varphi^4}\right] - 2a_1\frac{a'}{a}\frac{(\varphi')^3}{\varphi^4} + \\ + a_2[\frac{a'a''}{a^2}\frac{\varphi'}{\varphi^2} + \frac{a''}{a}\left(\frac{(\varphi')^2}{\varphi^3} - \frac{\varphi''}{\varphi^2}\right) - \frac{\varphi'}{\varphi^2}\frac{a'''}{a}] = 0 \qquad (4.16)$$

Solving these equations, the first we will do is to make the transformation to cosmological time (for a review of the method see [9]): $dz = a(y)dy$. After this, the equations (4.16) take the form:

$$
\begin{aligned}
&-4b[a^3 \ddddot{a} + 3a^2\dot a \dddot a + a^2(\ddot a)^2 - 5a(\dot a)^2\ddot a] + \\
&+24b' a\dot a^2 \ddot a - \frac{24}{\kappa}a^4\Lambda - \frac{12}{\kappa}\left(a^3\ddot a + a^2\dot a^2\right) + \\
&+a_1 a^4 \frac{\dot\varphi^4}{\varphi^4} + 2a_2 a^2[a^2\frac{\ddot\varphi^2}{\varphi^2} + \\
&+3a\dot a\frac{\dot\varphi\ddot\varphi}{\varphi^2} + a^2\frac{\dot\varphi\dddot\varphi}{\varphi^2} - 5a^2\frac{\dot\varphi^2\ddot\varphi}{\varphi^3} - 3a\dot a\frac{\dot\varphi^3}{\varphi^3} + \\
&+3a^2\frac{\dot\varphi^4}{\varphi^4}] = 0
\end{aligned}
$$

$$
\begin{aligned}
&6a_1 \ln a a^2\left(a\frac{\dot\varphi^4}{\varphi^5} - a\frac{\dot\varphi^2\ddot\varphi}{\varphi^4} - \frac{\dot\varphi^3}{\varphi^4}\right) - 2a_1 a^2\dot a\frac{\dot\varphi^3}{\varphi^4} + \\
&+a_2(a^2\ddot a\frac{\dot\varphi^2}{\varphi^3} - a^2\ddot a\frac{\ddot\varphi}{\varphi^2} + a\dot a^2\frac{\dot\varphi^2}{\varphi^3} - a\dot a^2\frac{\ddot\varphi}{\varphi^2} + \\
&-a^2\dddot a\frac{\dot\varphi}{\varphi^2} - 4a\dot a\ddot a\frac{\dot\varphi}{\varphi^2} - \dot a^3\frac{\dot\varphi}{\varphi^2}) = 0
\end{aligned} \tag{4.17}
$$

Here $\dot a = \mathrm{d}a/\mathrm{d}z$ and $\dot\varphi = \mathrm{d}\varphi/\mathrm{d}z$.

As we see these equations are too complicated to be solved analytically and that is why it is necessary to look for their solutions approximately. So let us look for following Ansatz:

$$
a(z) \simeq a_0\,\mathrm{e}^{Hz}\,, \qquad \varphi(z) \simeq \varphi_0\,\mathrm{e}^{-\alpha Hz}\,. \tag{4.18}
$$

Taking into account that the logarithmic term is too small as $\ln a \sim Hz$ and H is proportional to the Plank mass, we may neglect it. So after this we obtain from the second equation in (4.17) for α the following algebraic equation,

$$
2a_1\alpha^3 + 6a_2\alpha = 0\,, \tag{4.19}
$$

and the solutions for this equation have the form:

$$
\alpha_1 = 0\,, \quad \alpha_{2,3} = \pm\sqrt{-\frac{3a_2}{a_1}} = \pm 2i\,. \tag{4.20}
$$

The imaginary solutions are not interesting because, as it is well known, in that case the creation of an AdS Universe is not possible as it would be unstable. The trivial solution leads to the case when there is no dilaton. We may conclude that the quantum correction we have included in the action, with out dilatonic potential does not give any effects in the early AdS Universe.

2. Now let us choose the dilatonic potential in (3.9) to be of the form $V(\varphi) = \alpha\ln\varphi$. Performing similar analysis as in the case before, we obtain the equations of motion in terms of z in the following form.

$$
\begin{aligned}
&-4b[a^3 \ddddot{a} + 3a^2\dot a \dddot a + a^2(\ddot a)^2 - 5a(\dot a)^2\ddot a] + \\
&+24b' a\dot a^2 \ddot a - \frac{24}{\kappa}a^4\Lambda - \frac{12}{\kappa}\left(a^3\ddot a + a^2\dot a^2\right) + \\
&+a_1 a^4 \frac{\dot\varphi^4}{\varphi^4} + 2a_2 a^2[a^2\frac{\ddot\varphi^2}{\varphi^2} + \\
&+3a\dot a\frac{\dot\varphi\ddot\varphi}{\varphi^2} + a^2\frac{\dot\varphi\dddot\varphi}{\varphi^2} - 5a^2\frac{\dot\varphi^2\ddot\varphi}{\varphi^3} - 3a\dot a\frac{\dot\varphi^3}{\varphi^3} + \\
&+3a^2\frac{\dot\varphi^4}{\varphi^4}] + 4\alpha a^4\ln\varphi = 0
\end{aligned}
$$

$$
\begin{aligned}
&6a_1 \ln a a^2\left(a\frac{\dot\varphi^4}{\varphi^5} - a\frac{\dot\varphi^2\ddot\varphi}{\varphi^4} - \frac{\dot\varphi^3}{\varphi^4}\right) - 2a_1 a^2\dot a\frac{\dot\varphi^3}{\varphi^4} + \\
&+a_2(a^2\ddot a\frac{\dot\varphi^2}{\varphi^3} - a^2\ddot a\frac{\ddot\varphi}{\varphi^2} + a\dot a^2\frac{\dot\varphi^2}{\varphi^3} - a\dot a^2\frac{\ddot\varphi}{\varphi^2} + \\
&-a^2\dddot a\frac{\dot\varphi}{\varphi^2} - 4a\dot a\ddot a\frac{\dot\varphi}{\varphi^2} - \dot a^3\frac{\dot\varphi}{\varphi^2}) + \frac{\alpha}{2}\frac{a^3}{\varphi} = 0
\end{aligned} \tag{4.21}
$$

As it was done above we look for approximated solutions like

$$
a(z) \simeq a_0\,\mathrm{e}^{Hz}\,, \qquad \varphi(z) \simeq \varphi_0\,\mathrm{e}^{-\alpha Hz}\,. \tag{4.22}
$$

Substituting in the second of equations (4.21) and taking in it α_0 small enough, we obtain for α_0 the following algebraic equation,

$$
4\alpha_0 + 8(4\pi)^2\alpha = 0\,, \tag{4.23}
$$

and

$$
\alpha_0 = -32\pi^2\alpha\,. \tag{4.24}
$$

Now from the first equation of (4.21), neglecting the logarithmic term as above, for H^2 the following solution is obtained

$$
H^2 = -\frac{1}{2\kappa b'}\left(-1 + \sqrt{1 + 4\kappa b'\Lambda}\right)\,. \tag{4.25}
$$

This solution is always positive, and it is important to remark that in such approximation the solution for H^2 does not depend on the value of parameter α, and it means that the quantum correction does not enter into H^2. Thus, we have found a non imaginary scale factor for dilatonic AdS Universe and of course there occurs the possibility for quantum creation of dilatonic AdS Universe.

3. Let us see one more case with the dilatonic potential defined as $V(\varphi) = \alpha_1\varphi$. Substituting this form for the potential in (3.10) and performing similar analysis we obtain the following solution for α_0:

$$
\alpha_0 = \frac{a_0\alpha_1}{6\kappa a_2}\mathrm{e}^{Hz}\,, \tag{4.26}
$$

and finally for H^2 the solution is:

$$
H^2 = -\frac{1}{2\kappa b'}\left[-1 + \sqrt{1 + 4\kappa\Lambda b' + 4\kappa b'\alpha_1\varphi_0}\right]\,. \tag{4.27}
$$

This solution is positive, as the third term in the radical is quite small, so the obtained value for H^2 fills the condition to obtain a non imaginary scale factor and therefore the possibility for the quantum creation of dilatonic Ads Universe.

5. Summary

Summarizing, it was shown that without dilaton the quantum effects lead to the annihilation of AdS Universe. We have seen how the form of the dilatonic potential affects the quantum creation of AdS Universe. It was shown the role of dilaton coupled quantum matter to stabilization of AdS Universe.

Using anomaly induced effective action for dilaton coupled scalar and adding it to the classical gravity action, it was done the analysis of the equations of motion, for a number of dilatonic potentials. On the basis of this it was shown that the possibility of quantum creation of dilatonic AdS space occurs.

The interest to AdS space is caused mainly by AdS/CFT correspondence and nice supersymmetric properties of this background. There was recently much interest also to brane-world approach where observable universe is considered as brane in higher dimensional space. There exits some variant of brane-world which is induced by brane quantum effects (so-called Brane New World)(for its formulation see [12, 13]).

It would be very interesting to understand the role of choice of dilatonic potential to quantum creation of dilatonic de Sitter and Anti-de Sitter spaces in Brane New World scenario. This will be discussed elsewhere.

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